

APPENDIX II:

A SIMPLE PROOF OF THE EVANS LEMMA.

The Evans Lemma is the direct result of the tetrad postulate of differential geometry:

$$\boxed{D_\mu v_\lambda^a = \partial_\mu v_\lambda^a + \omega_{\mu b}^a v_\lambda^b - \Gamma_{\mu\lambda}^\nu v_\nu^a = 0} \quad - (1)$$

using the notation of the text.

It follows from eqn (1) that:

$$D^\mu (D_\mu v_\lambda^a) = \partial^\mu (D_\mu v_\lambda^a) = 0, \quad - (2)$$

i.e

$$\partial^\mu (D_\mu v_\lambda^a + \omega_{\mu b}^a v_\lambda^b - \Gamma_{\mu\lambda}^\nu v_\nu^a) = 0, \quad - (3)$$

or

$$\square v_\lambda^a = \partial^\mu (\Gamma_{\mu\lambda}^\nu v_\nu^a) - \partial^\mu (\omega_{\mu b}^a v_\lambda^b). \quad - (4)$$

Define:

$$R v_\lambda^a := \partial^\mu (\Gamma_{\mu\lambda}^\nu v_\nu^a) - \partial^\mu (\omega_{\mu b}^a v_\lambda^b) \quad - (5)$$

to state the Evans Lemma:

$$\boxed{\square v_\lambda^a = R v_\lambda^a} \quad - (6)$$